

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

9340929644

FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

October/November 2023

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Any blank pages are indicated.

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1 Show that the system of equation	ons
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$$14x-4y+6z = 5, x+y+kz = 3, -21x+6y-9z = 14,$$

where k is a constant, does not have a unique solution and interpret this situation geometrically. [4]

where $r > 0$ and 0 \cdot	< ∀ < ∠π.			
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4 F	ind the	particular	solution	of the	differential	equation
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$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = 27x^2,$$

given that, when $x = 0$, $y = 2$ and	$\frac{\mathrm{d}y}{\mathrm{d}x} = -8.$	[10]

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5	The curve	Chas	parametric	equations
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$x = \frac{2}{3}t^{\frac{3}{2}} - 2t^{\frac{1}{2}},$	y = 2t + 5,	for $0 < t \le 3$
$x - \frac{1}{3}i^{2} - 2i^{2}$	y-2i+3,	101 $0 < t \le 3$

nd the exact length of C .

	nd the set of values of t for whic	-			
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6	(a)	Starting from the definitions of cosh and sinh in terms of exponentials, prove that			
		$\sinh 2x = 2\sinh x \cosh x.$	[3]		
	(b)	Using the substitution $u = \sinh x$, find $\int \sinh^2 2x \cosh x dx$.	[4]		
		J			
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$\frac{\mathrm{d}y}{\mathrm{d}x} + y \tanh x = \sinh^2 2x,$					
given that $y = 4$ when $x = 0$. Give your answer in the form $y = f(x)$. [7]					
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7	The	matrix	Δ	10	orven	h
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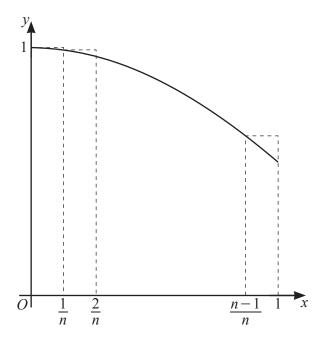
$$\mathbf{A} = \begin{pmatrix} -6 & 2 & 13 \\ 0 & -2 & 5 \\ 0 & 0 & 8 \end{pmatrix}.$$

Find a matrix P and a diagonal matrix D such that $\mathbf{A}^{-1} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$.	
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(b)	Use the characteristic equation of \mathbf{A} to find \mathbf{A}^{-1} .	[4]
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By letting $z = \cos \theta + i \sin \theta$, where $\cos \theta \neq 1$, show that	
$1 + \cos\theta + \cos 2\theta + \dots + \cos(n-1)\theta = \frac{1}{2} \left(1 - \cos n\theta + \frac{\sin n\theta \sin \theta}{1 - \cos \theta} \right).$	[7]

8



The diagram shows the curve with equation $y = \cos x$ for $0 \le x \le 1$, together with a set of *n* rectangles of width $\frac{1}{n}$.

(c) By considering the sum of the areas of these rectangles, show that

$\int_0^1 \cos x dx < \frac{1}{2n} \left(1 - \cos 1 + \frac{\sin 1 \sin \frac{1}{n}}{1 - \cos \frac{1}{n}} \right). $ [4]
Use a similar method to find, in terms of n , a lower bound for $\int_0^1 \cos x dx$. [3]

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Additional page

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